JEE MAINS-2018

IMPORTANT INSTRUCTIONS

1. Immediately fill in the particulars on this page of the Test Booklet with only Black Ball Point Pen provided in the examination hall.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of 3 hours duration.
4. The Test Booklet consists of 90 questions. The maximum marks are 360.
5. There are three parts in the question paper A, B, C consisting of **Physics, Mathematics and Chemistry** having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for each correct response.
6. Candidates will be awarded marks as started above in instruction No. 5 for correct response of each question. ¼ (one fourth) marks of the total marks allotted to the question (i.e. 1 mark) will be deducted for indicating incorrect response of each question. No deduction from that total score will be made if no response is indicated for an item in the answer sheet.
7. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. For writing particulars / marking responses on Side–1 and Side–2 of the Answer Sheet use only Black Ball Point Pen provided in the examination hall.
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination room/hall.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in four pages at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room / Hall. However, the candidates are allowed to take away this Test Booklet with them.
12. The CODE for this Booklet is D. Make sure that the CODE printed on Side–2 of the Answer Sheet is same as that on this Booklet. Also tally the serial number of Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
13. Do not fold or make any stray mark on the Answer Sheet. The test is of **3** hours duration.

PART-A-PHYSICS

1. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c, The values of p_d and p_c are respectively

(1) (0, 0) (2) (0, 1) (3) (·89, ·28) (4) (·28, ·89)

Ans. 3

Sol. Fraction loss in KE = $\dfrac{\frac{1}{2}m_2\left(\frac{2m_1u}{m_1+m_2}\right)^2}{\frac{1}{2}m_1u^2}$

2. The mass of a hydrogen molecule is 3.32 × 10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly:

(1) 2.35 × 10^2 N/m^2 (2) 4.70 × 10^2 N/m^2 (3) 2.35 × 10^3 N/m^2 (4) 4.70 × 10^3 N/m^2

Ans. 3

Sol. Pressure = $\dfrac{\text{Force}}{\text{area}} = \dfrac{\frac{\text{no. of collisions}}{\text{sec}} \times \frac{\text{change in momentum}}{\text{collision}}}{\text{Area}}$

$= \dfrac{10^{23} \times 2mv\cos 45°}{2 \times 10^{-4}}$ = 2.35 × 10^3 N/m^2

3. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$, is

(1) $\frac{mg}{3Ka}$ (2) $\frac{mg}{Ka}$ (3) $\frac{Ka}{mg}$ (4) $\frac{Ka}{3mg}$

Ans. 1

Sol. $k = \dfrac{\Delta P}{\Delta v / v} \Rightarrow k = \dfrac{\frac{mg}{a}}{3\frac{\Delta R}{R}}$ $\dfrac{\Delta R}{R} = \dfrac{mg}{3ka}$

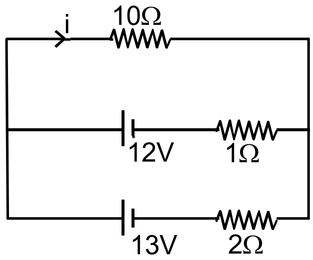
4. Two batteries with e.m.f. 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω respectively. The voltage across the load lies between:

(1) 11.4 V and 11.5 V (2) 11.7 V and 11.8 V

(3) 11.6 V and 11.7 V (4) 11.5 V and 11.6 V

Ans. 4

Sol. $v = i\,(10) = \frac{\varepsilon_{eq}}{R_{eq}}\,(10)$



$$v = \frac{\dfrac{\frac{12}{1}+\frac{13}{2}}{\frac{1}{1}+\frac{1}{2}}}{10+\frac{2}{3}} \times 10 = 11.56 \text{ volts}$$

5. A particle is moving in a circular path of radius a under the action of an attractive potential $U = \frac{-k}{2r^2}$. Its total energy is:

(1) zero (2) $\frac{-3k}{2a^2}$ (3) $\frac{-k}{4a^2}$ (4) $\frac{k}{2a^2}$

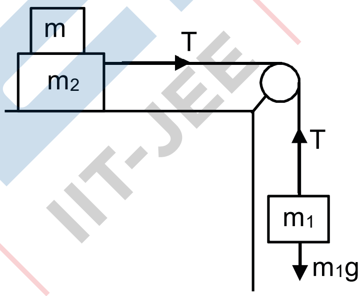
Ans. 1

Sol. $u = \frac{-k}{2r^2} \Rightarrow F = \frac{-dU}{dr} \Rightarrow F = \frac{k}{r^3}$

$$\frac{mv^2}{r} = \frac{k}{r^3} \Rightarrow v^2 = \frac{k}{mr^2}$$

$$T.E. = \frac{1}{2}mv^2 + P.E. = \frac{k}{2r^2} + \left(-\frac{k}{2r^2}\right) = 0$$

6. Two masses m_1 = 5 kg and m_2 = 10 kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is:



(1) 43.3 kg (2) 10.3 kg (3) 18.3 kg (4) 27.3 kg

Ans. 4

Sol. $\mu(m + m_2) = m_1$

$$m = \frac{m_1}{\mu} - m_2 = 23.33 \text{ kg}$$

7. If the series limit frequency of the Lyman series is v_L, then the series limit frequency of the Pfund series is:

(1) $\frac{v_L}{16}$ (2) $\frac{v_L}{25}$ (3) $25\,v_L$ (4) $16\,v_L$

Ans. 2

Sol. $\frac{1}{\lambda_{Lyman}} = R\left(\frac{1}{1^2} - \frac{1}{n_2^2}\right) \Rightarrow \frac{1}{\lambda_{Lyman}} = R \; \{n_2 = \infty\}$

$\frac{1}{\lambda_{pfund}} = R\left(\frac{1}{5^2} - \frac{1}{n_2^2}\right) = \frac{R}{25} \; \{n_2 = \infty\}$

$\frac{f_{Lyman}}{f_{pfund}} = 25 \Rightarrow f_{pfund} = \frac{v_L}{25}$

8. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is:

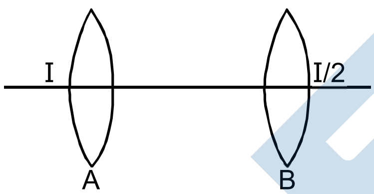
(1) 45° (2) 60° (3) 0° (4) 30°

Ans. 1

Sol. Intensity after a = $\frac{I}{2}$

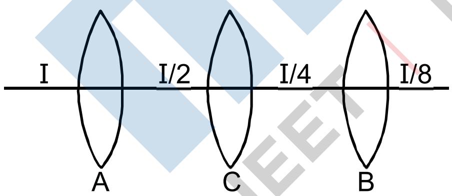
Intensity after b = $\frac{I}{2}$

so, transmission axis of a and b are parallel.



Use $I = I_0 \cos^2 \phi$

$\frac{I}{2} \cos^2 \phi \times \cos^2 \phi = \frac{I}{8}$



So, angle between A and C is 45°

9. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n, (A, B are constants)

(1) $\Lambda_n^2 \approx A + B\lambda_n^2$ (2) $\Lambda_n^2 \approx \lambda$ (3) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$ (4) $\Lambda_n \approx A + B\lambda_n$

Ans. 3

Sol. By rydberg's

$$\frac{1}{\Lambda_n} = R\left(1 - \frac{1}{n^2}\right)$$

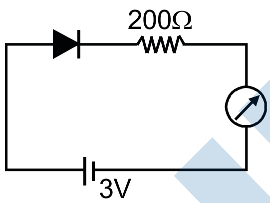
$$\Lambda_n = \frac{1}{R}\left(1 - \frac{1}{n^2}\right)^{-1} = \frac{1}{R}\left(1 + \frac{1}{n^2}\right) \qquad ......(1)$$

$$\because \lambda_n = \frac{2\pi r_n}{n} = 2\pi r_0 n = n\lambda_g \qquad ......(2)$$

By 1 and 2

$$\Lambda n = \frac{1}{R}\left(1 + \frac{\lambda_g^2}{\lambda_n^2}\right)$$

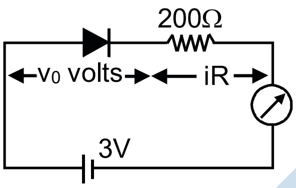
10. The reading of the ammeter for a silicon diode in the given circuit is:



(1) 11.5 mA (2) 13.5 mA (3) 0 (4) 15 mA

Ans. 1

Sol.



3 = 0.7 + i(200)

i = 11.5 mA

11. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is

(1) $r_e < r_p < r_\alpha$ (2) $r_e < r_\alpha < r_p$ (3) $r_e > r_p = r_\alpha$ (4) $r_e < r_p = r_\alpha$

Ans. 4

Sol. $R = \frac{mv}{qB}$ ......(1)

$$\frac{(P_e)^2}{2m_e} = \frac{(P_P)^2}{2m_p} = \frac{(P_\alpha)^2}{2m_\alpha} = \text{constant}$$

$P_e = \sqrt{2m_e \text{ const.}}$

$P_p = \sqrt{2m_p \text{const.}}$

$P_\alpha = \sqrt{2m_\alpha \text{ const.}}$

$$R = \frac{\sqrt{2m \text{ const.}}}{qB}$$

$R \propto \frac{\sqrt{m}}{q}$ $\qquad$ $r_e < r_p = r_\alpha$

12. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be

(1) 2.4 nC (2) 0.9 nC (3) 1.2 nC (4) 0.3 nC

Ans. 3

Sol. $Q_{ind} = Q\left(1-\frac{1}{k}\right) = 90 \times 10^{-12} \times 20 \times \frac{5}{3}\left(1-\frac{1}{5/3}\right) = 1.2 \text{ nC}$

13. For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by

(1) $\frac{R}{(\omega_0 C)}$ (2) $\frac{CR}{\omega_0}$ (3) $\frac{\omega_0 L}{R}$ (4) $\frac{\omega_0 R}{L}$

Ans. 3

Sol. Formula based

Q factor = $\frac{\omega_0 L}{R}$

14. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?

(1) 2×10^5 (2) 2×10^6 (3) 2×10^3 (4) 2×10^4

Ans. 1

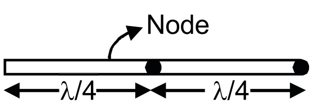
Sol. No. of telephonic channel = $\frac{10 \times 10^9 \times 0.1}{5 \times 10^3} = 2 \times 10^5$

15. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m^3 and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations?

(1) 10 kHz (2) 7.5 kHz (3) 5 kHz (4) 2.5 kHz

Ans. 3

Sol. $\frac{\lambda}{2} = 0.6 \text{ m} \Rightarrow \lambda = 1.2 \text{ m}$



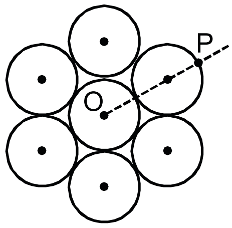
$v = f\lambda$

$\sqrt{\frac{Y}{\rho}} = f\lambda$

f = 4.882 kHz

$f \approx 5$ kHz

16. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is:



(1) $\frac{73}{2}MR^2$ (2) $\frac{181}{2}MR^2$ (3) $\frac{19}{2}MR^2$ (4) $\frac{55}{2}MR^2$

Ans. 2

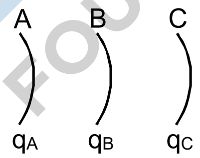
Sol. $I_p = \frac{MR^2}{2} + \left(6\left\{\frac{MR^2}{2} + M(2R)^2\right\}\right) + 7M(3R)^2 = \frac{181MR^2}{2}$

17. Three concentric metal shells A, B and C of respective radii a, b and c (a < b < c) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively. The potential of shell B is:

(1) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$ (2) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$ (3) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$ (4) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$

Ans. 4

Sol. $V_B = \frac{q_A}{4\pi\epsilon_0 b} + \frac{q_B}{4\pi\epsilon_0 b} + \frac{q_C}{4\pi\epsilon_0 C}$



$= \frac{\sigma(4\pi a^2)}{4\pi\epsilon_0 b} - \frac{\sigma(4\pi b^2)}{4\pi\epsilon_0 b} + \frac{\sigma(4\pi c^2)}{4\pi\epsilon_0 c}$

$= \frac{\sigma}{\epsilon_0}\left[\frac{a^2}{b} - b + c\right] = \frac{\sigma}{\epsilon_0}\left(\frac{a^2-b^2}{b}+c\right)$

18. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

(1) $2\ \Omega$ (2) $2.5\ \Omega$ (3) $1\ \Omega$ (4) $1.5\ \Omega$

Ans. 4

Sol. Standard Results :

$r_{in} = R_{sh}\left(\frac{\ell_1}{\ell_2} - 1\right)$

where ℓ_1 = Balance length without shunt

ℓ_2 = Balance length with shunt

$= 1.5\ \Omega$

19. An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01}\,\hat{x}\cos\left[2\pi v\left(\frac{z}{c} - t\right)\right]$ in air and $\vec{E}_2 = E_{02}\,\hat{x}\cos[k(2z - ct)]$ in medium, where the wave number k and frequency v refer to their values in air. The medium is non-magnetic. If ϵ_{r_1} and ϵ_{r_2} refer to relative permittivities of air and medium respectively, which of the following options is correct?

(1) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$ (2) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{2}$ (3) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 4$ (4) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 2$

Ans. 1

Sol. Wave speed = $\left|\frac{\text{coeff. of } t}{\text{coeff. of } z}\right|$

$\therefore V_{air} = C;\ V_{med} = \frac{C}{2}$

Now, $\frac{1}{\mu_0\ \epsilon_0\epsilon_{r_1}} \Big/ \frac{1}{\mu_0\ \epsilon_0\epsilon_{r_2}} = \frac{C^2}{4}$

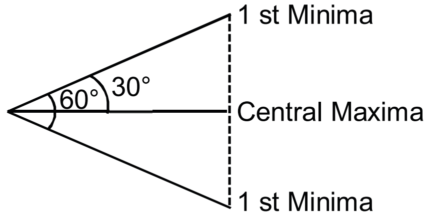
$\Rightarrow \frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$

20. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringes width is 1 cm, what is slit separation distance?
(i.e. distance between the centres of each slit.)

(1) 75 μm (2) 100 μm (3) 25 μm (4) 50 μm

Ans. 3

Sol.



$\therefore \sin 30° = \frac{\lambda}{a} \Rightarrow \lambda = 0.5\ \mu m$

Later,

$\beta = \frac{D\lambda}{d} \Rightarrow d = \frac{D\lambda}{\beta} = 25\ \mu m$

21. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other? (Mole weight of silver = 108 and Avogadro number = 6.02×10^{23} gm mole^{-1})

(1) 2.2 N/m (2) 5.5 N/m (3) 6.4 N/m (4) 7.1 N/m

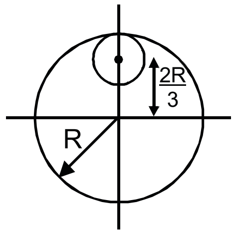
Ans. 4

Sol. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$k = 4\pi^2 f^2 m$

$= \frac{4\pi^2 \times 10^{24} \times 108 \times 10^{-3}}{6.02 \times 10^{23}}$ = 7.08 N/m

22. From a uniform circular disc of radius R and mass 9 M, a small disc of radius is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is



(1) $10\ MR^2$ (2) $\frac{37}{9}MR^2$ (3) $4\ MR^2$ (4) $\frac{40}{9}MR^2$

Ans. 3

Sol. $I = \frac{1}{2} \times 9MR^2 - \left[\frac{1}{2}M \times \frac{R^2}{9} + M \times \frac{4R^2}{9}\right]$

$= \left(\frac{9}{2} - \frac{1}{2}\right)MR^2$

$= 4MR^2$

23. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is:

(1) $\frac{v_0}{2}$ (2) $\frac{v_0}{\sqrt{2}}$ (3) $\frac{v_0}{4}$ (4) $\sqrt{2}\ v_0$

Ans. 4

Sol. $\frac{3}{2}\left(\frac{1}{2}mv_0^2\right) = \frac{(mv_0)^2}{4m} + \frac{1}{2}\frac{m}{2}v_{rel}^2$

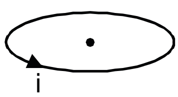
$v_{rel} = \sqrt{2}\ v_0$

24. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is

(1) $\sqrt{2}$ (2) $\frac{1}{\sqrt{2}}$ (3) 2 (4) $\sqrt{3}$

Ans. 1

Sol. Dipole moment = $\pi R^2 \times I$



$B_1 = \frac{\mu_0 I}{2R}$

$$B_2 = \frac{\mu_0 I}{2\sqrt{2}R}$$

$$\frac{B_1}{B_2} = \sqrt{2}$$

25. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is

(1) 4.5% (2) 6% (3) 2.5% (4) 3.5%

Ans. 1

Sol. $\rho = \frac{m}{\ell^3}$

$$\frac{d\rho}{\rho} = \frac{dm}{m} + 3 \times \frac{d\ell}{\ell}$$

26. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances?

(1) 550 Ω (2) 910 Ω (3) 990 Ω (4) 505 Ω

Ans. 1

Sol. $\frac{R_1}{R_2} = \frac{x}{100 - x}$

$$\frac{R_2}{R_1} = \frac{x-10}{100-(x-10)} = \frac{x-10}{110-x}$$

$$\left(\frac{x}{100-x}\right)\left(\frac{x-10}{110-x}\right) = 1$$

$x^2 - 10x = 11000 - 210x + x^2$

$200x = 11000$

$x = 55$

$R_1 = 550$

27. In an a.c. circuit, the instantaneous e.m.f. and current are given by

$e = 100 \sin 30t$

$i = 20 \sin\left(30t - \frac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively

(1) $\frac{50}{\sqrt{2}}$, 0 (2) 50, 0 (3) 50, 10 (4) $\frac{1000}{\sqrt{2}}$, 10

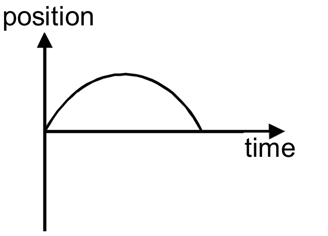
Ans. 4

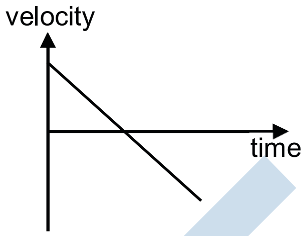
Sol. $p_{av} = \frac{e_0}{\sqrt{2}} \frac{i_0}{\sqrt{2}} \cos \Delta\phi$

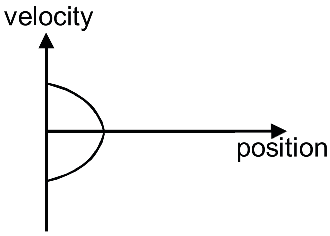
$$= \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1000}{\sqrt{2}}$$

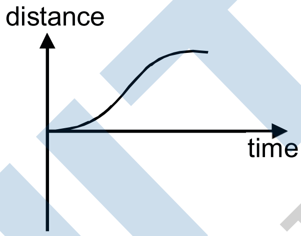
wattless current = $\frac{i_0 \cos\Delta\varphi}{\sqrt{2}}$

28. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

(A)


(B)


(C)


(D*)


Ans. 4

Sol. (1) and (2) Matches

first half of (1) is diff. than first half of (4)

29. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2 V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

(1) (a) 189 K, (b) – 2.7 kJ
(2) (a) 195 K, (b) 2.7 kJ
(3) (a) 189 K, (b) 2.7 kJ
(4) (a) 195 K, (b) – 2.7 kJ

Ans. 1

Sol. $(2v)T^{5/3-1}$ $(v)\ (273 + 27)^{5/3-1}$

T = 189 K

$\Delta U < 0$

30. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then

(1) $T \propto R^{\frac{(n+1)}{2}}$
(2) $T \propto R^{\frac{n}{2}}$
(3) $T \propto R^{\frac{3}{2}}$ for any n
(4) $T \propto R^{\frac{n}{2}+1}$

Ans. 1

Sol. $m\omega^2 R \propto \frac{1}{R^n}$

$\omega \propto \frac{1}{R^{\frac{n+1}{2}}}$

$T \propto R^{\frac{n+1}{2}}$

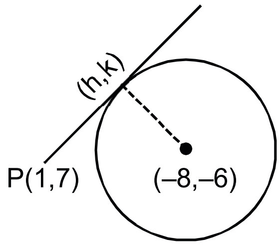
PART-B-MATHEMATICS

31. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is

(1) 85 (2) 95 (3) 195 (4) 185

Ans. 2

Sol. $x^2 = y - 6$



$2x = y'$

y' at (1, 7) = 2

$\frac{k-7}{h-1} = 2$

$\Rightarrow k - 7 = 2h - 2$

$\Rightarrow 2h - k = -5$ ....(i)

Now, $\frac{k+6}{h+8} = \frac{-1}{2}$

$\Rightarrow 2k + 12 = -h - 8$

$\Rightarrow h + 2k = -20$ ....(ii)

from (i) and (ii)

$h = -6$, $k = -7$

$x^2 + y^2 + 16x + 12y + c = 0 \Rightarrow 36 + 49 - 96 - 84 + c = 0 \Rightarrow c = 95.$

32. If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is :

(1) $\frac{1}{2\sqrt{2}}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{4\sqrt{2}}$ (4) $\frac{1}{3\sqrt{2}}$

Ans. 4

Sol. $2x - 2y + 3z - 2 = 0$ ....(i)

$x - y + z + 1 = 0$ ....(ii)

Solve (i) and (ii)

$z = 4, x = -5, y = 0$ (let)

$\therefore (-5, 0, 4)$

$$\vec{n}_1 = \vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i}(1) - \hat{j}(-1) + \hat{k}(0) = \hat{i} + \hat{j}$$

$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 3\hat{i} - 5\hat{j} - 7\hat{k}$$

plane containing vectors $\vec{n}_1 \& \vec{n}_2$ and also the point (–5, 0, 4)

$$\begin{vmatrix} x+5 & y & z-4 \\ 1 & 1 & 0 \\ 3 & -5 & -7 \end{vmatrix} = 0$$

$7x - 7y + 8z + 3 = 0$

Distance from the origin $d = \frac{3}{\sqrt{7^2+7^2+8^2}} = \frac{1}{3\sqrt{2}}$.

33. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to :

(1) 1 (2) 2 (3) –1 (4) 0

Ans. 1

Sol. $\alpha, \beta = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm i\sqrt{3}}{2}$

$\alpha = \frac{1+i\sqrt{3}}{2} = -\omega^2$

$\beta = \frac{1-i\sqrt{3}}{2} = -\omega$

$\alpha^{101} + \beta^{107} = (-\omega^2)^{101} + (-\omega)^{107} = -[\omega + \omega^2] = 1.$

34. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of ΔPTQ is :

(1) $60\sqrt{3}$ (2) $36\sqrt{5}$ (3) $45\sqrt{5}$ (4) $54\sqrt{3}$

Ans. 3

Sol. $\frac{x^2}{9} - \frac{y^2}{36} = 1$

$y = mx \pm \sqrt{9m^2 - 36}$

Tangent passes through (0, 3)

$3 = \pm \sqrt{9m^2 - 36}$

$9 = 9m^2 - 36$

$m = \pm \sqrt{5}$

Equation of PT

$y = \sqrt{5}x + 3 \Rightarrow \frac{-\sqrt{5}x}{3} + \frac{y}{3} = 1$ ....(1)

Let P be (x_1, y_1)

$\frac{xx_1}{9} - \frac{yy_1}{36} = 1$ ....(2)

Comparing (1) & (2)

$\frac{x_1}{9} = \frac{-\sqrt{5}}{3} \Rightarrow x_1 = -3\sqrt{5}$

$\frac{-y_1}{36} = \frac{1}{3} \Rightarrow y_1 = -12,$

$\therefore$ P $(-3\sqrt{5}, -12)$

$\therefore$ Required area $= \frac{1}{2} \times 15 \times 6\sqrt{5} = 45\sqrt{5}$.

35. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is:

(1) 4 (2) $\frac{9}{2}$ (3) 6 (4) $\frac{7}{2}$

Ans. 2

Sol. $y^2 = 6x$, $9x^2 + by^2 = 16$

$2y\,y' = 6$ $18x + 2by\,y' = 0$

$y' = \frac{3}{y_1}$ $y' = \frac{-9x_1}{by_1}$

$\frac{3}{y_1} \times \left(\frac{-9x_1}{by_1}\right) = -1$

$27x_1 = b\,y_1^2$

$27x_1 = b.\,6x_1$

$b = \frac{9}{2}$ $\{\because x_1 \neq 0\}$.

36. If the system of linear equations

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to :

(1) –30 (2) 30 (3) –10 (4) 10

Ans. 4

Sol. $x + ky + 3z = 0$ ....(1)

$3x + ky - 2z = 0$ ....(2)

$2x + 4y - 3z = 0$ ....(3)

(1) – (2) $\Rightarrow 2x = 5z$

Put in (3) $\Rightarrow 2z + 4y = 0,\ z = -2y$

$$\therefore \frac{xz}{y^2} = \frac{\frac{5z}{2}\cdot z}{\frac{z^2}{4}} = 10.$$

37. Let S = $\{x \in R : x \geq 0 \text{ and } 2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0\}$. Then S :

(1) contains exactly two elements. (2) contains exactly four elements.

(3) is an empty set. (4) contains exactly one element.

Ans. 1

Sol. $2\left|\sqrt{x}-3\right|+x-6\sqrt{x}+6=0$

$2\left|\sqrt{x}-3\right|+\left(\sqrt{x}-3\right)^2=3$ $\qquad\qquad \left|\sqrt{x}-3\right|=t$

$t^2 + 2t - 3 = 0$

$(t + 3)(t - 1) = 0$

$t = -3$ (reject), 1

$\left|\sqrt{x}-3\right|=1 \Rightarrow \sqrt{x}-3=\pm1 \Rightarrow \sqrt{x}=4, 2 \Rightarrow x = 16, 4.$

38. If sum of all the solutions of the equation $8\cos x\left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0, \pi]$ is $k\pi$, then k is equal to :

(1) $\frac{8}{9}$ (2) $\frac{20}{9}$ (3) $\frac{2}{3}$ (4) $\frac{13}{9}$

Ans. 4

Sol. $8\cos x\left(\cos^2\frac{\pi}{6}-\sin^2 x-\frac{1}{2}\right)=1$

$8\cos x\left(\frac{1}{4}-\sin^2 x\right)=1$

$2\cos x\,(1 - 4\sin^2 x) = 1$

$2\cos x - 4\sin x\,.\,\sin 2x = 1$

$2\cos x - 2(\cos x - \cos 3x) = 1$

$\cos 3x = \frac{1}{2}$

$3x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$

$\Rightarrow x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$

required same $= \frac{13\pi}{9}$.

39. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is :

(1) $\frac{1}{5}$ (2) $\frac{3}{4}$ (3) $\frac{3}{10}$ (4) $\frac{2}{5}$

Ans. 4

Sol. Bag → 4 red, 6 black

Required probability = $\frac{4}{10}\times\frac{6}{12}+\frac{6}{10}\times\frac{4}{12}=\frac{48}{10\times12}=\frac{2}{5}$.

40. Let f(x) = $x^2 + \frac{1}{x^2}$ and g(x) = $x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$. If h(x) = $\frac{f(x)}{g(x)}$, then the local minimum value of h (x) is :

(1) $-2\sqrt{2}$ (2) $2\sqrt{2}$ (3) 3 (4) –3

Ans. 2

Sol. $h(x) = \frac{f(x)}{g(x)} = \frac{x^2+\frac{1}{x^2}}{x-\frac{1}{x}}$ $x-\frac{1}{x}=t$

$= \frac{t^2+2}{t}$

$h(x) = t + \frac{2}{t}$

$h(x) \geq 2\sqrt{2}$

Point of local minimum value is $2\sqrt{2}$.

41. Two sets A and B are as under:

A = $\{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\}$;

B = $\{(a, b) \in R \times R: 4(a - 6)^2 + 9(b - 5)^2 \leq 36\}$. Then:

(1) $A \cap B = \phi$ (an empty set) (2) neither $A \subset B$ nor $B \subset A$

(3) $B \subset A$ (4) $A \subset B$

Ans. 4

Sol. A = shaded region

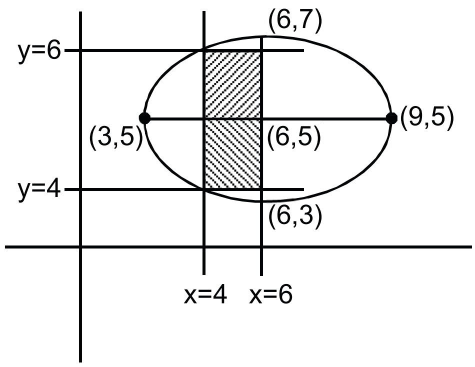
A = $\{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\}$;

$\Rightarrow -1 < a - 5 < 1 \Rightarrow 4 < a < 6 \ \& \ 4 < b < 6$

B = inner part of ellipse

B = $4(a - 6)^2 + 9(b - 5)^2 \leq 36$

$\Rightarrow \frac{(a-6)^2}{9}+\frac{(b-5)^2}{4}\leq 1$

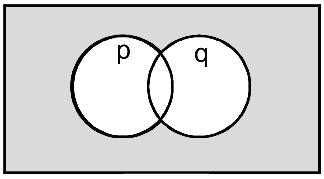


Clearly $A \subset B$.

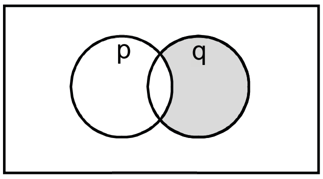
42. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to:

(1) q (2) ~ q (3) ~ p (4) p

Ans. 3

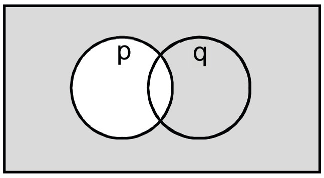


$\sim p \vee q$



$\sim p \wedge q$

Sol. $\therefore \sim (p \vee q) \vee (\sim p \wedge q) = \sim p$

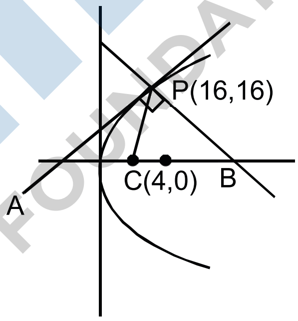


43. Tangent and normal are drawn at P(16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is:

(1) 3 (2) $\frac{4}{3}$ (3) $\frac{1}{2}$ (4) 2

Ans. 4

Sol. Tangent at P



$y.16 = \frac{16(x+16)}{2}$

$\Rightarrow 2y = x + 16$

$\therefore A = (-16, 0)$

and Normal at P

$y - 16 = -2(x - 16)$

$\Rightarrow 2x + y = 48$

$\therefore B = (24, 0)$

Clearly AB will be diameter of circle

$\therefore C = (4, 0)$

$\therefore$ slope of CP = $\frac{16-0}{16-4} = \frac{16}{12} = \frac{4}{3} = m_1$

and slope of PB = $m_2 = -2$

$$\therefore \tan\theta = \left|\frac{\frac{4}{3}+2}{1+\frac{4}{3}(-2)}\right| = \frac{10}{5} = 2$$

44. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$, then the ordered pair (A, B) is equal to

(1) (–4, 5) (2) (4, 5) (3) (–4, –5) (4) (–4, 3)

Ans. 1

Sol. $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3$

$\begin{vmatrix} 5x-4 & 5x-4 & 5x-4 \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}$

Applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$

$\begin{vmatrix} 0 & 0 & 5x-4 \\ x+4 & x+4 & 2x \\ 0 & x+4 & x-4 \end{vmatrix} = (5x-4)(x+4)^2$

$\therefore$ A = –4 and B = 5

$\therefore$ (A, B) = (–4, 5).

45. The sum of the co-efficient of all odd degree terms in the expansion of

$\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$, $(x > 1)$ is:

(1) 1 (2) 2 (3) –1 (4) 0

Ans. 2

Sol Let $y = \sqrt{x^3-1}$

$(x + y)^5 = {}^5C_0x^5 + {}^5C_1x^4y + {}^5C_2\,x^3y^2 \ldots\ldots + {}^5C_5y^5$

& $(x - y)^5 = {}^5C_0x^5 - {}^5C_1x^4y + {}^5C_2x^3y^2 \ldots\ldots - {}^5C_5y^5$

$\therefore (x + y)^5 + (x - y)^5 = 2\,({}^5C_0\,x^5 + {}^5C_2x^3y^2 + {}^5C_4\,xy^4) = 2\,(x^5 + 10\,x^3\,(x^3 - 1) + 5\,x\,(x^3 - 1)^2)$

$= 2\,(x^5 + 10\,x^6 - 10x^3 + 5\,x\,(x^6 - 2x^3 + 1))$

$\therefore$ Sum of coefficients = 2 (1 – 10 + 5 + 5) = 2.

46. Let $a_1, a_2, a_3, \ldots\ldots\ldots\ldots, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$.

If $a_1^2 + a_2^2 + \ldots\ldots\ldots\ldots + a_{17}^2 = 140\,m$, then m is equal to:

(1) 34 (2) 33 (3) 66 (4) 68

Ans. 1

Sol. $\sum_{K=0}^{12} a_{4K+1} = 416$

Let first term be a & common different be d

$\therefore a_1 + a_5 + \ldots\ldots\ldots$ 13 terms = 416

$\Rightarrow \frac{13}{2}\,(2a + (13-1)\,4d) = 416$

$\Rightarrow 13\,(a + 24\,d) = 416 \Rightarrow a + 24\,d = 32$ ....(1)

$\because a_9 + a_{43} = a + 8d + a + 42\,d \Rightarrow 2a + 50\,d = 66$

$a + 25d = 33$ ....(2)

By solving (1) & (2) $\Rightarrow$ d = 1 and a = 8

$\therefore a_1^2 + a_2^2 + ...a_{17}^2 = 8^2 + 9^2 + ... + 24^2 = (1^2 + 2^2 + ... + 24^2) - (1^2 + 2^2 + ... + 7^2)$

$= \frac{24 \times 25 \times 49}{6} - \frac{7 \times 8 \times 15}{6} = 4900 - 140 = 140(35 - 1) \Rightarrow m = 34.$

47. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P & Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is:

(1) $3x + 2y = xy$ (2) $3x + 2y = 6xy$ (3) $3x + 2y = 6$ (4) $2x + 3y = xy$

Ans. 1

Sol. Equation of PQ is $\frac{x}{h} + \frac{y}{k} = 1$

(2, 3) is on it

$\therefore \frac{2}{h} + \frac{3}{k} = 1 \Rightarrow 3h + 2k = hk$

$\therefore$ Locus is $3x + 2y = xy$.

48. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is:

(1) 4π (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{8}$ (4) $\frac{\pi}{2}$

Ans. 2

Sol. $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ ....(1)

Applying King

$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2^x \sin^2 x}{2^x + 1} dx$ ....(2)

From (1) + (2)

$2I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \, dx = 2\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$

$\therefore I = \frac{\pi}{4}$.

49. Let $g(x) = \cos x^2$ $\sqrt{x}$, f(x) = and α, β ($\alpha < \beta$) be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve y = (gof) (x) and the lines $x = \alpha$, $x = \beta$ and y = 0, is

(1) $\frac{1}{2}(\sqrt{3} - \sqrt{2})$ (2) $\frac{1}{2}(\sqrt{2} - 1)$ (3) $\frac{1}{2}(\sqrt{3} - 1)$ (4) $\frac{1}{2}(\sqrt{3} + 1)$

Ans. 3

Sol. $\because\ 18x^2 - 9\pi x + \pi^2 = 0$

$18x^2 - 6\pi x - 3\pi x + \pi^2 = 0$

$\Rightarrow 6x(3x - \pi) - \pi(3x - \pi) = 0 \Rightarrow (3x - \pi)(6x - \pi) = 0$

$\Rightarrow x = \frac{\pi}{3}$ or $\frac{\pi}{6}$

$\therefore \alpha = \frac{\pi}{6}, \beta = \frac{\pi}{3}$

gof (x) = cos x

$\Rightarrow$ Area bounded $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos x\, dx = (\sin x)_{\pi/6}^{\pi/3} = \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2}$.

50. For each $t \in R$, let [t] be the greatest integer less than or equal to t. Then

$$\lim_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \text{..........} + \left[\frac{15}{x}\right]\right)$$

(1) is equal to 120

(2) does not exist (in R)

(3) is equal to 0

(4) is equal to 15

Ans. 1

Sol. $\lim_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \text{..........} + \left[\frac{15}{x}\right]\right)$

$x\left(\frac{1}{x} - 1 + \frac{2}{x} - 1 + \ldots.. + \frac{15}{x} - 1\right) <$ Given expression $\le x\left(\frac{1}{x} + \frac{2}{x} + \ldots.. + \frac{15}{x}\right)$

$\therefore$ Limit = 1 + 2 + 3 + ... + 15 = $\frac{15 \times 16}{2}$ = 120.

51. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, \ldots\ldots, x_9$ is:

(1) 2

(2) 3

(3) 9

(4) 4

Ans. 1

Sol. var.(x) = var.(x – 5) = $\frac{1}{9}\sum_{i=1}^{9}(x_i - 5)^2 - \left(\frac{1}{9}\sum_{i=1}^{9}(x_i - 5)\right)^2$ = 5 – 1 = 4

$\therefore$ standard deviation = 2.

52. The integral $\int \frac{\sin^2 x \cos^2 x}{\left(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x\right)^2} dx$ is equal to

(1) $\frac{1}{1 + \cot^3 x} + C$

(2) $\frac{-1}{1 + \cot^3 x} + C$

(3) $\frac{1}{3\,(1 + \tan^3 x)} + C$

(4) $\frac{-1}{3\,(1 + \tan^3 x)} + C$

(where C is a constant of integration)

Ans. 4

Sol. $\int \frac{\sin^2 x \cos^2 x}{(\sin^3 x(\sin^2 x + \cos^2 x) + \cos^3 x(\sin^2 x + \cos^2 x)^2} dx$

$$= \int \frac{\sin^2 x \cos^2 x}{(\sin^3 x + \cos^3 x)^2} dx = \int \frac{\tan^2 x \sec^2 x}{(\tan^3 x + 1)^2} dx$$

Let $\tan^3 x + 1 = t$

$\Rightarrow 3\tan^2 x \sec^2 x\, dx = dt$

$$\therefore\ I = \int \frac{\frac{1}{3} dt}{t^2} = -\frac{1}{3t} + C = -\frac{1}{3(\tan^3 x + 1)} + C$$

53. Let S = {t ∈ R : f(x) = |x – π| · ($e^{|x|}$ –1) sin | x | is not differentiable at t}. Then the set S is equal to

(1) {π} (2) {0, π} (3) φ (an empty set) (4) {0}

Ans. 3

Sol. f(x) = |x – π| ($e^{|x|}$ – 1) sin |x|

Clearly derivable at π as well as 0

∴ derivable everywhere.

54. Let y = y(x) be the solution of the differential equation $\sin x \frac{dy}{dx} + y\cos x = 4x, x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to:

(1) $-\frac{8}{9}\pi^2$ (2) $-\frac{4}{9}\pi^2$ (3) $\frac{4}{9\sqrt{3}}\pi^2$ (4) $\frac{-8}{9\sqrt{3}}\pi^2$

Ans. 1

Sol. $\sin x \frac{dy}{dx} + y \cos x = 4x$

$\Rightarrow \frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x$

I.F. = $e^{\int \cot x dx} = e^{\ln \sin x} = \sin x$

∴ solution is

$y \times \sin x = \int 4x\, dx = 2x^2 + C$

$\therefore x = \frac{\pi}{2}, y = 0 \Rightarrow C = -\frac{\pi^2}{2}$

$\therefore y \sin x = 2x^2 - \frac{\pi^2}{2}$

At $x = \frac{\pi}{6}$

$\frac{y}{2} = \frac{\pi^2}{18} - \frac{\pi^2}{2} \Rightarrow y = \frac{-8}{9}\pi^2.$

55. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. if $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}.\vec{b} = 24$, then $|\vec{u}|^2$ is equal to:

(1) 256 (2) 84 (3) 336 (4) 315

Ans. 3

Sol. $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}, \vec{b} = \hat{j} + \hat{k}$

$\vec{u} = x\vec{a} + y\vec{b}$

$\therefore\ \vec{u}\cdot\vec{a} = 0 \Rightarrow x\vec{a}\cdot\vec{a} + y\vec{a}\cdot\vec{b} = 0$

$\Rightarrow 14x + 2y = 0$

$\Rightarrow y = -7x$

and $\vec{u}\cdot\vec{b} = x\vec{a}\cdot\vec{b} + y\vec{b}\cdot\vec{b} = 24$

$\Rightarrow 2x + 2y = 24$

$\Rightarrow x + y = 12$

$\Rightarrow x - 7x = 12 \Rightarrow x = -2$ and $y = 14$

$\therefore \vec{u} = -2\vec{a} + 14\vec{b} = -2(2\hat{i} + 3\hat{j} - \hat{k}) + 14(\hat{j} + \hat{k}) = -4\hat{i} + 8\hat{j} + 16\hat{k}$

$\therefore\ |\vec{u}|^2 = 16 + 64 + 256 = 336.$

56. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, x + y + z = 7 is

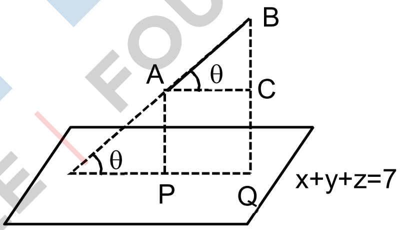
(1) $\frac{1}{3}$ (2) $\sqrt{\frac{2}{3}}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\frac{2}{3}$

Ans. 2

Sol. A(5, –1, 4), B = (4, –1, 3)

AB = $-\hat{i} - \hat{k}$

normal to plane = $\vec{n} = \hat{i} + \hat{j} + \hat{k}$



angle between line and plane is

$$\sin\theta = \frac{\vec{AB}\cdot\vec{n}}{|\vec{AB}||\vec{n}|} = \frac{-1-1}{\sqrt{2}\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\therefore \cos\theta = \sqrt{1 - \frac{2}{3}} = \frac{1}{\sqrt{3}}$$

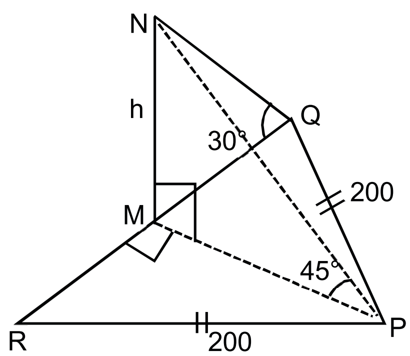
$\therefore$ projection = PQ = AB cos θ = $\frac{\sqrt{2}}{\sqrt{3}}$

57. PQR is a triangular park with PQ = PR = 200m. A T.V. tower stands at the mid point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is:

(1) $100\sqrt{3}$ (2) $50\sqrt{2}$ (3) 100 (4) 50

Ans. 3

Sol. Let height of tower be h



$\therefore$ QM = h cot 30° = $\sqrt{3}$ h

and PM = h cot 45° = h

$\therefore PQ^2 = PM^2 + QM^2$

$(200)^2 = 3h^2 + h^2$

h = 100 m.

58. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is:

(1) at least 500 but less than 750
(2) at least 750 but less than 1000
(3) at least 1000
(4) less than 500

Ans. 3

Sol 6N and 3D

no. of ways = $^6C_4 \times {}^3C_1 \times 4!$

$= 15 \times 3 \times 24 = 1080.$

59. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series $1^2 + 2\cdot2^2 + 3^2 + 2\cdot4^2 + 5^2 + 2\cdot6^2 + \ldots\ldots.$ If $B - 2A = 100\lambda$, then λ is equal to:

(1) 464
(2) 496
(3) 232
(4) 248

Ans. 4

Sol. $\therefore 1^2 + 2.2^2 + 3^2 + 2.4^2 + \ldots\ldots.. + 2.(2n)^2$

$\Rightarrow (1^2 + 2^2 + 3^2 \ldots\ldots\ldots.. (2n)^2 + (2^2 + 4^2 + 6^2 \ldots\ldots.. (2n)^2)$

$= = \dfrac{2n(2n+1)(4n+1)}{6} + \dfrac{4n(n+1)(2n+1)}{6} = \dfrac{2n(2n+1)}{6}(4n + 1 + 2n + 2) = \dfrac{2n(2n+1)(6n+3)}{6}$

$= n(2n + 1)^2$

$\therefore B - 2A = 20(41)^2 - 2 \times 10(21)^2$

$= 20(41^2 - 21^2) = 20(41 + 21)(41 - 21)$

$= 400 \times 62 = 100 \times 248$

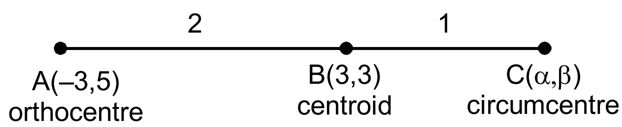
$\therefore \lambda = 248.$

60. Let the orthocentre and centroid of a triangle be A(–3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is:

(1) $3\sqrt{\dfrac{5}{2}}$
(2) $\dfrac{3\sqrt{5}}{2}$
(3) $\sqrt{10}$
(4) $2\sqrt{10}$

Ans. 1

Sol. $\therefore \dfrac{2\alpha + 1(-3)}{3} = 3 \Rightarrow \alpha = 6$



and $\dfrac{2\beta + 1\times5}{3} = 3 \Rightarrow \beta = 2$

$\therefore C = (6, 2)$

$\therefore AC = \sqrt{9^2 + 3^2} = \sqrt{81+9} = \sqrt{90} = 3\sqrt{10}$

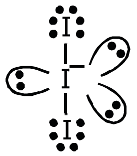
$\therefore$ radius $= \dfrac{3\sqrt{10}}{2} = 3\sqrt{\dfrac{5}{2}}$.

PART-C-CHEMISTRY

61. Total number of lone pair of electrons in I_3^- ion is :

(1) 9 (2) 12 (3) 3 (4) 6

Ans. 1

Sol. 

Total number of lone pair = 9

62. Which of the following salts is the most basic in aqueous solution?

(1) $FeCl_3$ (2) $Pb(CH_3COO)_2$ (3) $Al(CN)_3$ (4) CH_3COOK

Ans. 4

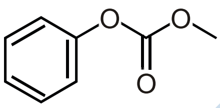
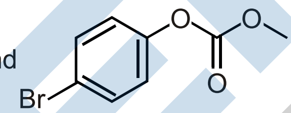
Sol. (1) $FeCl_3$ $\rightarrow$ Salt of Strong acid & weak base

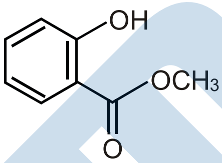
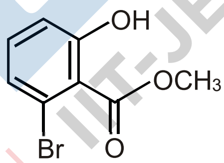
(2) $Pb(CH_3COO)_2$ $\rightarrow$ Salt of Weak acid & weak base

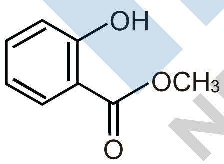
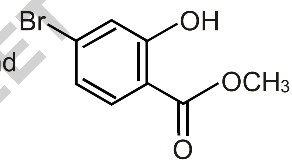
(3) $Al(CN)_3$ $\rightarrow$ Salt of Weak acid & weak base

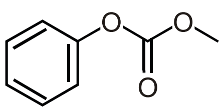
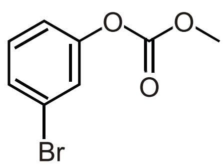
(4) CH_3COOK $\rightarrow$ Salt of Weak acid & Strong base therefore it is most basic in aqueous solution

63. Phenol reacts with methyl chloroformate in the presence of NaOH to form product A. A reacts with Br_2 to form product B. A and B are respectively:

(A)  and 

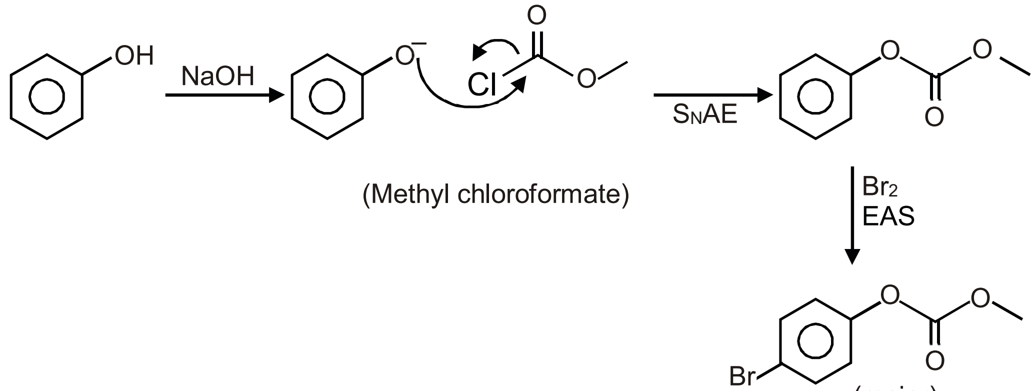
(B)  and 

(C)  and 

(D)  and 

Ans. 1

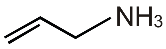
Sol.

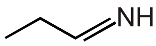


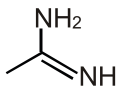
(Methyl chloroformate)

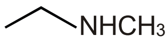
(major)

64. The increasing order of basicity of the following compounds is :

(a) 

(b) 

(c) 

(d) 

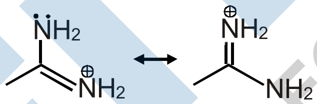
(1) (b) < (a) < (d) < (c)
(2) (d) < (b) < (a) < (c)
(3) (a) < (b) < (c) < (d)
(4) (b) < (a) < (c) < (d)

Ans. 1

Sol. (a) – sp^3 N atom with 1° amine

(b) – sp^2 N atom, thus less basic than a

(c) 2 equivalent RS in conjugate acid make it the most basic amongst given compounds.



(d) sp^3 N atom with 2° amine, thus more basic than (a)

65. An alkali is titrated against an acid with methyl orange as indicator, which of the following is a correct combination?

	Base	Acid	End point
(1)	Weak	Strong	Yellow to pinkish red
(2)	Strong	Strong	Pink to colourless
(3)	Weak	Strong	Colourless to pink
(4)	Strong	Strong	Pinkish red to yellow

Ans. 1

Sol. MeOH (aq) $\rightleftharpoons$ Me^+ (aq) + OH^- (aq)
Yellow pinkish red
(basic medium) (acidic medium)

In given titration, acid is being added into base means initially base is present so colour will be yellow which will turn into red at the end point.

66. The trans-alkenes are formed by the reduction of alkynes with:

(1) Na/liq.NH_3
(2) Sn-HCl
(3) H_2-Pd/C, $BaSO_4$
(4) $NaBH_4$

Ans. 1

Sol. Na/liq.NH_3 gives trans alkene with alkyne while H_2/Pd-$BaSO_4$ gives cis Alkene

67. The ratio of mass percent of C and H of an organic compound ($C_XH_YO_Z$) is 6 : 1. If one molecule of the above compound ($C_XH_YO_Z$) contains half as much oxygen as required to burn one molecule of compound C_XH_Y completely to CO_2 and H_2O. The empirical formula of compound $C_XH_YO_Z$ is :

(1) $C_3H_4O_2$ (2) $C_2H_4O_3$ (3) $C_3H_6O_3$ (4) C_2H_4O

Ans. 2

Sol. $C_XH_YO_Z$ Given

Ratio of C : H $C_XH_Y + \frac{1}{2}\left(2x+\frac{y}{2}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$

mass 6 : 1 $z = \frac{1}{2}\times\frac{1}{2}\left(2x+\frac{y}{2}\right)\times 2$

moles $\frac{6}{12}:\frac{1}{1}$ $4z = \left(2x+\frac{y}{2}\right)\times 2$

moles 1 : 2 $2Z = 2x + x$

$y = 2x$ $2Z = 3x$

$C_xH_{2x}O_z$

$C_xH_{2x}O_{\frac{3x}{2}} \rightarrow C_2H_4O_3$

68. Hydrogen peroxide oxidises $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN)_6]^{3-}$ to $[Fe(CN)_6]^{4-}$ in alkaline medium. The other products formed are, respectively:

(1) H_2O and $(H_2O + O_2)$ (2) H_2O and $(H_2O + OH^-)$

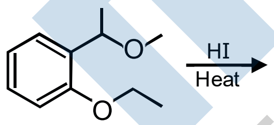
(3) $(H_2O + O_2)$ and H_2O (4) $(H_2O + O_2)$ and $(H_2O + OH^-)$

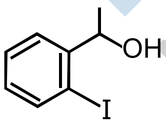
Ans. 1

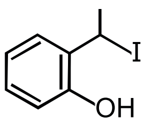
Sol. $2[Fe(CN)_6]^{4-} + 2H^+ + H_2O_2 \longrightarrow 2[Fe(CN)_6]^{3-} + 4H_2O$

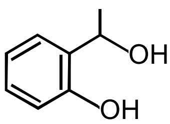
$2[Fe(CN)_6]^{3-} + 1H_2O_2 + 2OH^- \longrightarrow 2[Fe(CN)_6]^{4-} + O_2 + 2H_2O$

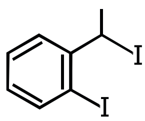
69. The major product formed in the following reaction is :



(1) 

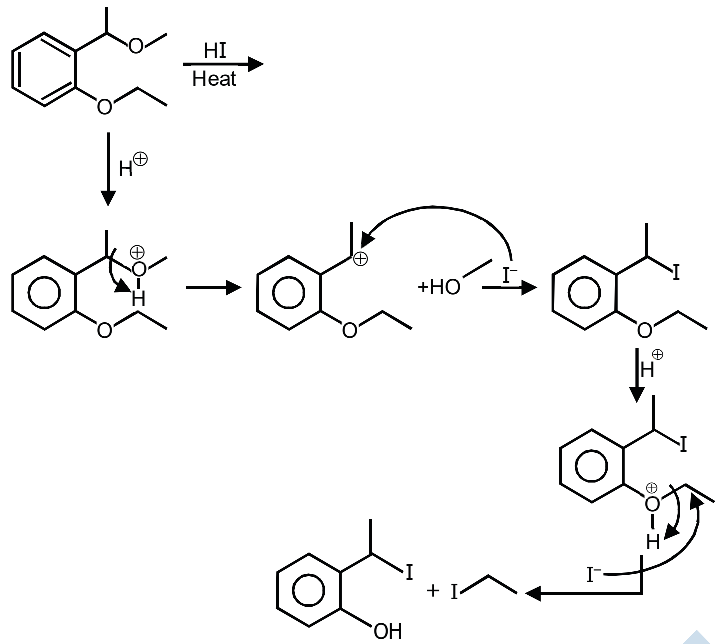
(2) 

(3) 

(4) 

Ans. 2

Sol.



70. How long (approximate) should water be electrolysed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane?
[Atomic weight of B = 10.8 u]

(1) 3.2 hours (2) 1.6 hours (3) 6.4 hours (4) 0.8 hour

Ans. 1

Sol. $B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$

1 mol 3 mol

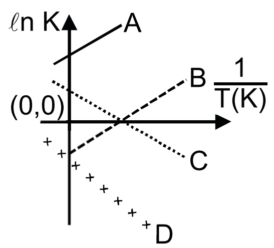
$2H_2O\,(l) \longrightarrow O_2\,(g) + 4H^+\,(aq) + 4e^-$

moles of $O_2 \times 4 = \dfrac{100 \times t}{96500}$

$t = \dfrac{96500 \times 3 \times 4}{3600}$

$\dfrac{965 \times 12}{3600} = 3.2$ hours

71. Which of the following lines correctly show the temperature dependence of equilibrium constant, K, for an exothermic reaction?



(1) C and D (2) A and D (3) A and B (4) B and C

Ans. 3

Sol. $\ln(K) = \dfrac{-\Delta H^\circ}{RT} + \dfrac{-\Delta S^\circ}{R}$

For exothermic reaction $\Delta H^\circ < 0$

so slope will be +ve

A and B are correct

72. At 518°C, the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 Torr, was 1.00 Torr s^{-1} when 5% has reacted and 0.5 Torr s^{-1} when 33% had reacted. The order of the reaction is :

(1) 1 (2) 0 (3) 2 (4) 3

Ans. 3

Sol. Rate = $k(P)^n$

$1 = k\,(0.95\,P_0)^n$

$0.5 = k\,(0.67\,P_0)^n$

$$\Rightarrow \frac{1}{0.5} = \left(\frac{0.95P_0}{0.67P_0}\right)^n$$

$\Rightarrow 2 = (1.4)^n$

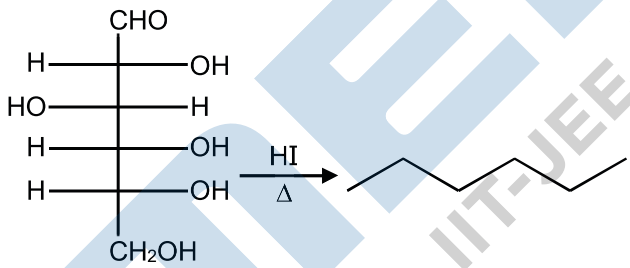
$\Rightarrow 2' = (2^{1/2})^n \Rightarrow n = 2$

73. Glucose on prolonged heating with HI gives:

(1) Hexanoic acid (2) 6-iodohexanal (3) n-Hexane (4) 1-Hexene

Ans. 3

Sol. Glucose $\xrightarrow[\Delta]{HI}$ n-Hexane, here HI acts as a reducing agent



74. Consider the following reaction and statements:

$[Co(NH_3)_4Br_2]^+ + Br^- \longrightarrow [Co(NH_3)_3Br_3] + NH_3$

(I) Two isomers are produced if the reactant complex ion is a cis-isomer.

(II) Two isomers are produced if the reactant complex ion is a trans-isomer.

(III) Only one isomer is produced if the reactant complex ion is a trans-isomer.

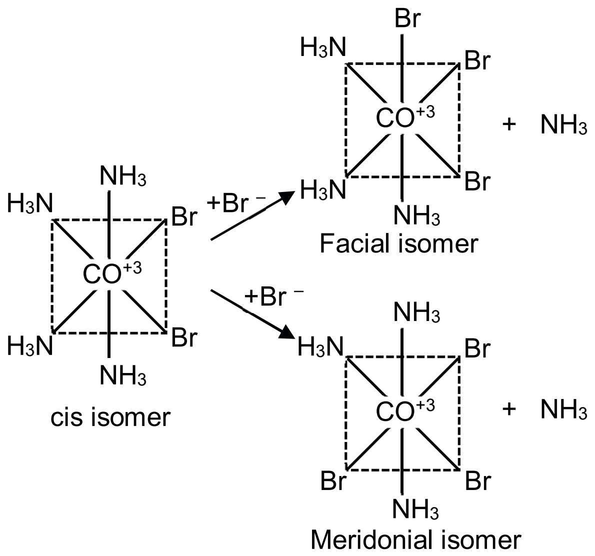
(IV) Only one isomer is produced if the reactant complex ion is a cis-isomer.

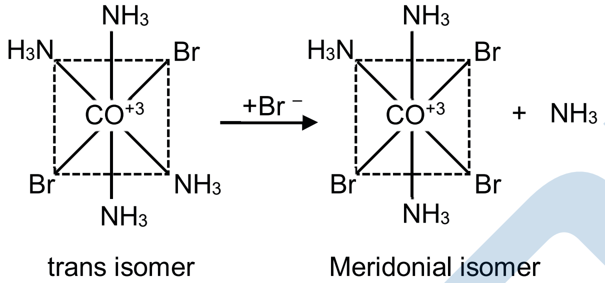
The correct statements are:

(1) (III) and (IV) (2) (II) and (IV) (3) (I) and (II) (4) (I) and (III)

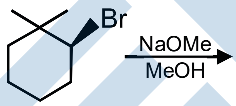
Ans. 4

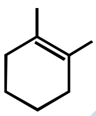
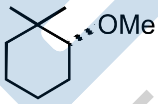
Sol. $[Co(NH_3)_4Br_2]^+ + Br^- \longrightarrow [Co(NH_3)_3Br_3] + NH_3$

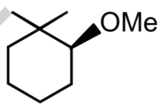


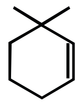


75. The major product of the following reaction is :



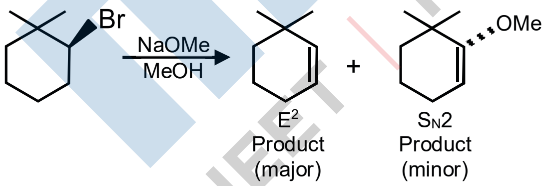
(1)  (2) 

(3) 

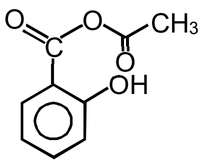
(4) 

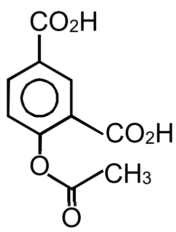
Ans. 4

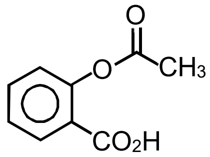
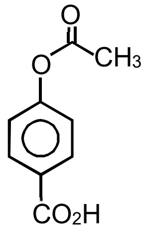
Sol.



76. Phenol on treatment with CO_2 in the presence of NaOH followed by acidification produces compound X as the major product. X on treatment with $(CH_3CO)_2O$ in the presence of catalytic amount of H_2SO_4 produces:

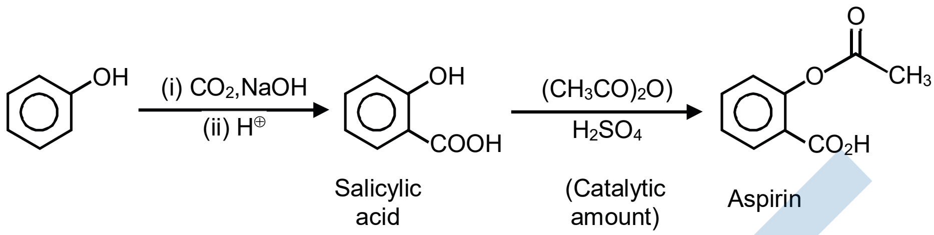
(1) 

(2) 

(3)    (4) 

Ans. 3

Sol.



77. An aqueous solution contains an unknown concentration of Ba^{2+}. When 50 mL of a 1 M solution of Na_2SO_4 is added, $BaSO_4$ just begins to precipitate. The final volume is 500 mL. The solubility product of $BaSO_4$ is 1×10^{-10}. What is the original concentration of Ba^{2+}?

(1) 1.1×10^{-9} M (2) 1.0×10^{-10} M (3) 5×10^{-9} M (4) 2×10^{-9} M

Ans. 1

Sol. Final volume = 500 ml

volume of Na_2SO_4 = 50 ml, so volume of Ba^{2+} solution = 450 mL

Let $[Ba^{2+}]_{original}$ = x M

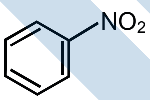
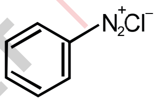
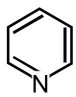
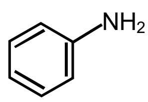
To begin precipitation of $BaSO_4$,

after mixing $[Ba^{2+}].[SO_4^{2-}] = K_{sp}$

$$\left(\frac{x \times 450}{500}\right).\left(\frac{1 \times 50}{500}\right) = 10^{-10}$$

$x = 1.1 \times 10^{-9}$ M

78. Which of the following compounds will be suitable for Kjeldahl's method for nitrogen estimation?

(1)  (2)  (3)  (4) 

Ans. 4

Sol. Kjeldahl's method is not applicable to compounds containing N in nitro, N is azo and N present in the ring (example : Pyridine)

79. When metal 'M' is treated with NaOH, a white gelatinous precipitate 'X' is obtained which is soluble in excess of NaOH. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal 'M' is :

(1) Al (2) Fe (3) Zn (4) Ca

Ans. 1

Sol. Al metal dissolves in NaOH & Al_2O_3 is used as adsorbent in chromatography so answer is (1)

80. An aqueous solution contains 0.10 M H_2S and 0.20 M HCl. If the equilibrium constant for the formation of HS^- from H_2S is 1.0×10^{-7} and that of S^{2-} from HS^- ions is 1.2×10^{-13} then the concentration of S^{2-} ions in aqueous solution is:

(1) 6×10^{-21} (2) 5×10^{-19} (3) 5×10^{-8} (4) 3×10^{-20}

Ans. 4

Sol. $k_1 = 10^{-7}$

$$H_2S \rightleftharpoons H^+ + HS^-$$

$0.1 - x$ $\quad 0.2 + x + y \approx 0.2$ $\quad x - y = x$

$k_2 = 1.2 \times 10^{-13}$

$$HS^- \rightleftharpoons H^+ + S^{2-}$$

$x - y \approx x$ $\quad 0.2 + x + y \approx 0.2$ $\quad y$

$10^{-7} = \dfrac{x \times 0.2}{0.1}$ $\quad x = 5 \times 10^{-8}$

$1.2 \times 10^{-13} = \dfrac{0.2 \times y}{5 \times 10^{-8}}$

$y = 3 \times 10^{-20} = [S^{2-}]$

81. The recommended concentration of fluoride ion in drinking water is up to 1 ppm as fluoride ion is required to make teeth enamel harder by converting $[3Ca_3(PO_4)_2 \cdot Ca(OH)_2]$ to :

(1) $[3Ca_3(PO_4)_2 \cdot CaF_2]$ (2) $[3\{Ca(OH)_2\} \cdot CaF_2]$

(3) $[CaF_2]$ (4) $[3(CaF_2) \cdot Ca(OH)_2]$

Ans. 1

Sol. $3Ca_3(PO_4)_2 \cdot Ca(OH)_2] \xrightarrow{F^-} [3Ca_3(PO_4)_2 \cdot CaF_2$

82. The compound that does not produce nitrogen gas by the thermal decomposition is :

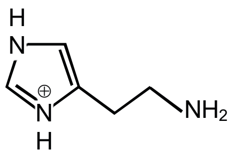
(1) NH_4NO_2 (2) $(NH_4)_2SO_4$ (3) $Ba(N_3)_2$ (4) $(NH_4)_2Cr_2O_7$

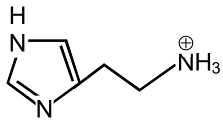
Ans. 2

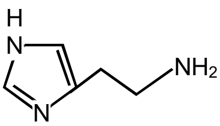
Sol. (1) $NH_4NO_2 \longrightarrow N_2\,(g) + 2H_2O$ (2) $(NH_4)_2SO_4 \longrightarrow NH_3\,(g) + H_2SO_4$

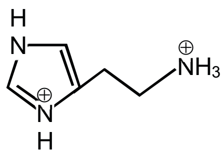
(3) $Ba(N_3)_2 \longrightarrow Ba(s) + N_2(g)$ (4) $(NH_4)_2Cr_2O \longrightarrow N_2\,(g) + 2H_2O + Cr_2O_3$

83. The predominant form of histamine present in human blood is (pK_a, Histidine = 6.0)

(1) 

(2) 

(3) 

(4) 

Ans. 2

Sol. Only the more basic atom will protonate.

84. The oxidation states of Cr in $[Cr(H_2O)_6]Cl_3$, $[Cr(C_6H_6)_2]$, and $K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$ respectively are :

(1) +3, 0 and +6 (2) +3, 0 and +4 (3) +3, +4 and +6 (4) +3, +2 and +4

Ans. 1

Sol. $[Cr(H_2O)_6]Cl_3 \longrightarrow Cr^{+3}$

$[Cr(C_6H_6)_2] \longrightarrow Cr^{0}$

$K_2[Cr(CN)_2(O^{-2})_2(O_2)^{-2}(NH_3)] \longrightarrow Cr^{+6}$

85. Which type of 'defect' has the presence of cations in the interstitial sites?

(1) Frenkel defect (2) Metal deficiency defect

(3) Schottky defect (4) Vacancy defect

Ans. 1

86. The combustion of benzene (l) gives CO_2(g) and H_2O(l). Given that heat of combustion of benzene at constant volume is –3263.9 kJ mol^{-1} at 25°C; heat of combustion (in kJ mol^{-1}) of benzene at constant pressure will be : [R = 8.314 JK^{-1} mol^{-1}]

(1) 3260 (2) –3267.6 (3) 4152.6 (4) –452.46

Ans. 2

Sol. ∵ Heat at constant volume ⇒ ΔU (Given)

and heat at constant pressure ⇒ ΔH (Asked)

For a reaction,

$$\Delta H = \Delta U + \Delta n_g.RT$$

Combustion reaction of $C_6H_6\ (\ell)$

$$C_6H_6\ (\ell) + \frac{15}{2}O_2(s) \longrightarrow 6\ CO_2\ (s) + 3H_2O\ (l)$$

$$\Delta H = (-3263.9\ kJ) + \frac{(-1.5)\times 8.314\times 298}{1000} kJ$$

$= -\ 3267.6$ kJ

87. Which of the following are Lewis acids?

(1) PH_3 and $SiCl_4$ (2) BCl_3 and $AlCl_3$ (3) PH_3 and BCl_3 (4) $AlCl_3$ and $SiCl_4$

Ans. 2 & 4

Sol. PH_3 is not a Lewis acid

While in BCl_3, $AlCl_3$, $SiCl_4$ central atom (more electropositive atom) contain vaccant orbital, hence they act as Lewis acid.

88. Which of the following compounds contain(s) no covalent bond(s) ?

$KCl, PH_3, O_2, B_2H_6, H_2SO_4$

(1) KCl (2) KCl, B_2H_6 (3) KCl, B_2H_6, PH_3 (4) KCl, H_2SO_4

Ans. 1

Sol. Only KCl is ionic and others are covalent molecules.

89. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point?

(1) $[Co(H_2O)_4Cl_2]Cl.2H_2O$ (2) $[Co(H_2O)_3Cl_3].3H_2O$

(3) $[Co(H_2O)_6]Cl_3$ (4) $[Co(H_2O)_5Cl]Cl_2.H_2O$

Ans. 2

Sol. $\because$ Freezing point of solution $(T_f) = T_f^0 - \Delta T_f$

so for maximum $T_f \Rightarrow \Delta T_f$ must be minimum

$\because \Delta T_f = K_f .m.i$

where $K_f \rightarrow$ Constant

$m \rightarrow$ Molality (same for all)

so for minimum $\Delta T_f \Rightarrow$ i should be minimum which is for $[Co(H_2O)_3Cl_3].3H_2O$

90. According to molecular orbital theory, which of the following will not be a viable molecule?

(1) H_2^- (2) H_2^{2-} (3) He_2^{2+} (4) He_2^+

Ans. 2

Sol.

	H_2^-	H_2^{2-}	He_2^{2+}	He_2^+
Bond order	0.5	0	1	0.5

Species having bond order zero does not exist (not a viable molecule).